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On the reactions $\mathrm{p + p \to p + \Lambda + K^+}$ and $\mathrm{p + p \to p + \Sigma^0 + K^+}$ near thresholds

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Abstract. The cross-sections for the reactions of the strange production $\mathrm{p + p \to p + \Lambda + K^+}$ and $\mathrm{p + p \to p + \Sigma^0 + K^+}$ near thresholds of the final states $\mathrm{p\Lambda K^+}$ and $\mathrm{p\Sigma^0 K^+}$ are calculated in the effective Lagrangian approach. Our approach is based on the dominant contribution of the one-pion exchange and strong interaction of the colliding protons in the initial state. The theoretical values of the cross-sections agree reasonably well with the experimental data. The polarization properties of the Λ- and Σ^0-hyperons are discussed.

PACS. 11.10.Cd Field theory: Axiomatic approach – 25.10.+s Nuclear reactions involving few-nucleon systems – 25.40.-h Nucleon-induced reactions – 25.40.Cm Elastic proton scattering

1 Introduction

Recent experimental data [1–4] on the production of strangeness in pp collisions, $\mathrm{p + p \to p + \Lambda + K^+}$ [1–4] and $\mathrm{p + p \to p + \Sigma^0 + K^+}$ [4], for energies of colliding protons in the region near the thresholds of the final states $\mathrm{p\Lambda K^+}$ and $\mathrm{p\Sigma^0 K^+}$ have represented experimental values of the cross-sections $\sigma^{\mathrm{pp\to p\Lambda K^+}}(\varepsilon)$ and $\sigma^{\mathrm{pp\to p\Sigma^0 K^+}}(\varepsilon)$ with high precision, where ε is an excess of energy that we define below. As has been obtained in ref. [4] the cross-section for the Σ^0-hyperon, $\mathrm{p + p \to p + \Sigma^0 + K^+}$, exceeds by a factor 28 the cross-section for the production of Λ-hyperon, $\mathrm{p + p \to p + \Lambda + K^+}$, measured for the equivalent excess energies. These data give a possibility for testing of various theoretical approaches to mechanisms of a strangeness production from nucleons that is important for correct description of a strangeness production in heavy-ion collisions.

The parer is organized as follows. In section 1 we calculate the effective Lagrangians of the transitions $\mathrm{p + p \to p + \Lambda + K^+}$ and $\mathrm{p + p \to p + \Sigma^0 + K^+}$ in the one-pion exchange approximation and at leading order in momentum expansion in powers of the momenta of final-state particles. In section 2 we calculate the cross-sections for the reactions $\mathrm{p + p \to p + \Lambda + K^+}$ and $\mathrm{p + p \to p + \Sigma^0 + K^+}$ and compare the theoretical results with the experimental data. In the conclusion we discuss the obtained results and the polarization properties of the Λ^0- and Σ^0-hyperons.

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2 Effective Lagrangians of transitions $\mathrm{p + p \to p + \Lambda(\Sigma^0) + K^+}$

In our approach to the description of the reactions $\mathrm{p + p \to p + \Lambda + K^+}$ and $\mathrm{p + p \to p + \Sigma^0 + K^+}$, first, we suggest to investigate the transitions $\mathrm{p + p \to p + \Lambda + K^+}$ and $\mathrm{p + p \to p + \Sigma^0 + K^+}$, where the wave functions of the protons in the initial pp state are described by plane waves and all particles in the final states $\mathrm{p\Lambda K^+}$ and $\mathrm{p\Sigma^0 K^+}$ are decoupled. These transitions we define by the effective Lagrangians $\mathcal{L}^{\mathrm{pp\to p\Lambda K^+}}(x)$ and $\mathcal{L}^{\mathrm{pp\to p\Sigma^0 K^+}}(x)$. For the evaluation of these effective Lagrangian we suggest to use a simplest one-pion exchange approximation. The Feynman diagrams defining the effective Lagrangians $\mathcal{L}^{\mathrm{pp\to p\Lambda K^+}}(x)$ and $\mathcal{L}^{\mathrm{pp\to p\Sigma^0 K^+}}(x)$ are depicted in fig. 1.

The analytical expressions corresponding to these diagrams read

$$\begin{aligned}
&M(\mathrm{pp \to p\Lambda K^+}) = \\
&= [\bar{u}(\mathbf{p}_{\mathrm{p}}, \alpha_{\mathrm{p}}) i\gamma^5 u(\mathbf{p}_1, \alpha_1)] \,\frac{g^2_{\pi NN}}{M^2_\pi - (p_{\mathrm{p}} - p_1)^2} \\
&\times [\bar{u}(\mathbf{p}_\Lambda, \alpha_\Lambda) i\gamma^5 \,\frac{g_{p\Lambda K^+}}{M_{\mathrm{p}} - \hat{p}_\Lambda - \hat{p}_K}\, i\gamma^5 u(\mathbf{p}_2, \alpha_2)] \\
&-[\bar{u}(\mathbf{p}_{\mathrm{p}}, \alpha_{\mathrm{p}}) i\gamma^5 u(\mathbf{p}_2, \alpha_2)] \,\frac{g^2_{\pi NN}}{M^2_\pi - (p_{\mathrm{p}} - p_2)^2} \\
&\times [\bar{u}(\mathbf{p}_\Lambda, \alpha_\Lambda) i\gamma^5 \,\frac{g_{p\Lambda K^+}}{M_{\mathrm{p}} - \hat{p}_\Lambda - \hat{p}_K}\, i\gamma^5 u(\mathbf{p}_1, \alpha_1)], \qquad (1)
\end{aligned}$$

where $g_{\pi\mathrm{NN}} = 13.4$ is the coupling constant of the πNN interaction, $u(\mathbf{p}_i, \alpha_i)$ are bispinors of the protons for $i =$

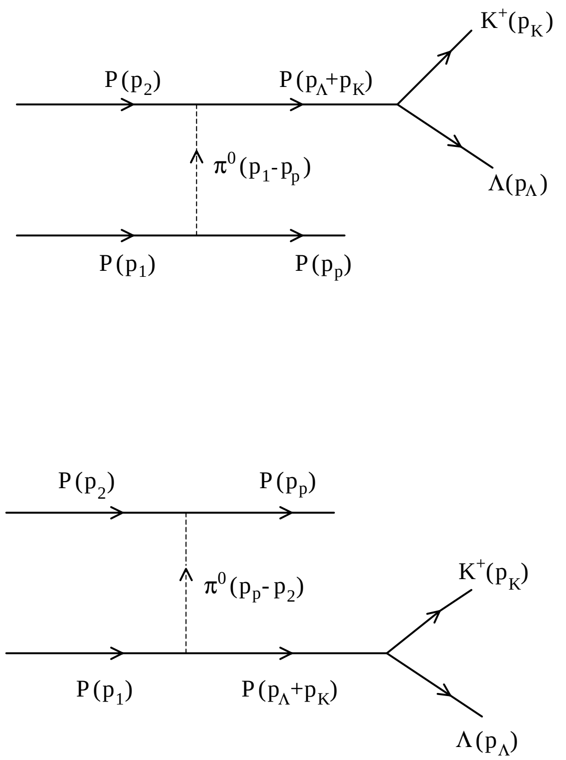

Fig. 1. The one-pion exchange diagrams describing the effective Lagrangian of the low-energy transition p + p → p + Λ + K^+.

$1, 2, 3$ and the Λ-hyperon for $i = \Lambda$ with polarizations α_i. Then, $M_\pi = 135\,\mathrm{MeV}$ and $M_\mathrm{p} = 938.3\,\mathrm{MeV}$ are the masses of the π^0-meson and the proton. The amplitude of the transition p + p → p + Σ^0 + K^+ can obtained from eq. (1) by the replacements $g_{p\Lambda K^+} \to g_{p\Sigma^0 K^+}$ and $p_\Lambda \to p_{\Sigma^0}$. We would like to accentuate that we are using the pseudoscalar couplings for the description of the π^0 and the K^+-meson coupled to baryons that always fit data well [5].

In the center-of-mass frames of the colliding protons and the pΛ system the amplitude eq. (1) takes the form

see equation (2) *on the next page*

where $\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}$ is a relative momentum of the colliding protons, $\mathbf{q}_{\mathrm{p}\Lambda}$ is a relative momentum of the pΛ system, and $\mathbf{p}_\mathrm{K}$ is the momentum of the K^+-meson.

The reaction p + p → p + $\Lambda(\Sigma^0)$ + K^+ is determined experimentally very close to threshold of the final state pΛK^+ (or p$\Sigma^0 K^+$). The minimum relative 3-momentum of the initial protons is equal to $|\mathbf{p}\,|_{\mathrm{threshold}} = p_0 = \sqrt{(M_\Lambda + M_{K^+} - M_\mathrm{p})(M_\Lambda + M_{K^+} + 3M_\mathrm{p})}/2 = 861.6\,\mathrm{MeV}$, where we have used $M_\Lambda = 1115.7\,\mathrm{MeV}$ and $M_{K^+} = 493.7\,\mathrm{MeV}$, the masses of the Λ-hyperon and the K^+ meson [6][1]. Due to this close vicinity to threshold the momentum of the K^+-meson and the relative movement of the pΛ system (or pΣ^0) are smaller compared with all energy scales of the coupled particles. This allows to expand the matrix element eq. (2) in powers of $\mathbf{p}_\mathrm{K}$ and $\mathbf{q}_{\mathrm{p}\Lambda}$ by keep leading contributions:

$$\begin{aligned}
&M(\mathrm{pp} \to \mathrm{p}\Lambda\mathrm{K}^+) = \\
&-\frac{g_{\mathrm{p}\Lambda\mathrm{K}^+} g^2_{\pi\mathrm{NN}}}{M_\mathrm{p} + M_\Lambda + M_{\mathrm{K}^+}}\,\frac{1}{M^2_\pi + 2M_\mathrm{p}(\sqrt{M^2_\mathrm{p} + p^2_0} - M_\mathrm{p})} \\
&\times \{[\bar{u}(-\mathbf{q}_{\mathrm{p}\Lambda} - \mathbf{p}_\mathrm{K}/2, \alpha_\mathrm{p}) i\gamma^5 u(\mathbf{p}, \alpha_1)] \\
&\times [\bar{u}(\mathbf{q}_{\mathrm{p}\Lambda} - \mathbf{p}_\mathrm{K}/2, \alpha_\Lambda) u(-\mathbf{p}, \alpha_2)] \\
&- \bar{u}(-\mathbf{q}_{\mathrm{p}\Lambda} - \mathbf{p}_\mathrm{K}/2, \alpha_\mathrm{p}) i\gamma^5 u(-\mathbf{p}, \alpha_2)] \\
&\times [\bar{u}(\mathbf{q}_{\mathrm{p}\Lambda} - \mathbf{p}_\mathrm{K}/2, \alpha_\Lambda) u(\mathbf{p}, \alpha_1)]\}. \qquad (3)
\end{aligned}$$

By introducing the effective coupling constant $C_{\mathrm{p}\Lambda\mathrm{K}^+}$

$$C_{\mathrm{p}\Lambda\mathrm{K}^+} = \frac{g_{\mathrm{p}\Lambda\mathrm{K}^+} g^2_{\pi\mathrm{NN}}}{M_\mathrm{p} + M_\Lambda + M_{\mathrm{K}^+}} \times \frac{1}{M^2_\pi + 2M_\mathrm{p}(\sqrt{M^2_\mathrm{p} + p^2_0} - M_\mathrm{p})}, \qquad (4)$$

we can write down the effective Lagrangian $\mathcal{L}^{\mathrm{pp}\to\mathrm{p}\Lambda\mathrm{K}^+}(x)$ of the transition p + p → p + Λ + K^+. That reads

$$\begin{aligned}
&\mathcal{L}^{\mathrm{pp}\to\mathrm{p}\Lambda\mathrm{K}^+}(x) = \\
&\quad - C_{\mathrm{p}\Lambda\mathrm{K}^+}\,\varphi^\dagger_{\mathrm{K}^+}(x)\,[\bar{p}(x) i\gamma^5 p(x)][\bar{\Lambda}(x) p(x)], \qquad (5)
\end{aligned}$$

where $p(x)$ and $\Lambda(x)$ are the operators of the interpolating proton and Λ-hyperon fields, and $\varphi^\dagger_{\mathrm{K}^+}(x)$ is the operator of the interpolating K^+-meson field. By making the replacement $\Lambda \to \Sigma^0$ in eq. (5) we obtain the effective Lagrangian $\mathcal{L}^{\mathrm{pp}\to\mathrm{p}\Sigma^0\mathrm{K}^+}(x)$ of the transition p + p → p + Σ^0 + K^+.

It is convenient to represent the effective Lagrangian in terms of the interactions describing the pΛ system in the certain spin state. This can be carried out by means of a Fierz transformation [7]. By performing a Fierz transformation, we recast the effective Lagrangian $\mathcal{L}^{\mathrm{pp}\to\mathrm{p}\Lambda\mathrm{K}^+}(x)$ into the form

$$\begin{aligned}
&\mathcal{L}^{\mathrm{pp}\to\mathrm{p}\Lambda\mathrm{K}^+}(x) = i\,\frac{1}{4}\,C_{\mathrm{p}\Lambda\mathrm{K}^+}\,\varphi^\dagger_{\mathrm{K}^+}(x) \\
&\times \{[\bar{p}(x)\gamma^5 \Lambda^c(x)][\bar{p^c}(x) p(x)] + [\bar{p}(x)\Lambda^c(x)][\bar{p^c}(x)\gamma^5 p(x)] \\
&+ [\bar{p}(x)\gamma^\mu \Lambda^c(x)][\bar{p^c}(x)\gamma_\mu\gamma^5 p(x)]\}. \qquad (6)
\end{aligned}$$

The first term and the last two in the Lagrangian equation (6) describe the pΛ system coupled in the spin singlet and triplet state, respectively.

Since near threshold the pΛ (or pΣ^0) system couples mainly in the spin singlet state, ${}^1S_0$, we should leave only the first term. This yields the following effective Lagrangian:

$$\begin{aligned}
\mathcal{L}^{\mathrm{pp}\to\mathrm{p}\Lambda\mathrm{K}^+}(x) &= i\,\frac{1}{4}\,C_{\mathrm{p}\Lambda\mathrm{K}^+}\,\varphi^\dagger_{\mathrm{K}^+}(x) \\
&\quad \times [\bar{p}(x)\gamma^5 \Lambda^c(x)][\bar{p^c}(x) p(x)]. \qquad (7)
\end{aligned}$$

[1] For the reaction p+p→p+Σ^0+K^+ the minimum relative momentum of the colliding protons amounts to $|\mathbf{p}|_{\mathrm{threshold}} = p_0 = \sqrt{(M_{\Sigma^0} + M_{K^+} - M_\mathrm{p})(M_{\Sigma^0} + M_{K^+} + 3M_\mathrm{p})}/2 = 917.5\mathrm{MeV}$ at $M_{\Sigma^0} = 1192.6\mathrm{MeV}$ [6].

$$
\begin{aligned}
&M(\mathrm{pp}\to \mathrm{p}\Lambda\mathrm{K}^+) = [\bar{\mathrm{u}}(-\mathbf{q}_{\mathrm{p}\Lambda}-\mathbf{p}_{\mathrm{K}}/2,\alpha_{\mathrm{p}})\mathrm{i}\gamma^5\mathrm{u}(\mathbf{p},\alpha_1)]\\
&\times \frac{g^2_{\pi NN}}{M^2_\pi-\left(\sqrt{M^2_{\mathrm{p}}+(\mathbf{q}_{\mathrm{p}\Lambda}+\mathbf{p}_{\mathrm{K}}/2)^2}-\sqrt{M^2_{\mathrm{p}}+\mathbf{p}^{\,2}}\right)^2+(\mathbf{p}+\mathbf{q}_{\mathrm{p}\Lambda}+\mathbf{p}_{\mathrm{K}}/2)^2}\\
&\times[\bar{u}(\mathbf{q}_{\mathrm{p}\Lambda}-\mathbf{p}_{\mathrm{K}}/2,\alpha_\Lambda)i\gamma^5\\
&\times\frac{g_{p\Lambda K^+}}{M_{\mathrm{p}}-\gamma^0\left(\sqrt{M^2_\Lambda+(\mathbf{q}_{\mathrm{p}\Lambda}-\mathbf{p}_{\mathrm{K}}/2)^2}+\sqrt{M^2_{\mathrm{K}}+\mathbf{p}^2_{\mathrm{K}}}\right)+\gamma\cdot(\mathbf{q}_{\mathrm{p}\Lambda}+\mathbf{p}_{\mathrm{K}}/2)}\,i\gamma^5u(-\mathbf{p},\alpha_2)]\\
&-[\bar{u}(-\mathbf{q}_{\mathrm{p}\Lambda}-\mathbf{p}_{\mathrm{K}}/2,\alpha_{\mathrm{p}})i\gamma^5u(-\mathbf{p},\alpha_2)]\\
&\times \frac{g^2_{\pi NN}}{M^2_\pi-\left(\sqrt{M^2_{\mathrm{p}}+(\mathbf{q}_{\mathrm{p}\Lambda}+\mathbf{p}_{\mathrm{K}}/2)^2}-\sqrt{M^2_{\mathrm{p}}+\mathbf{p}^{\,2}}\right)^2+(\mathbf{p}-\mathbf{q}_{\mathrm{p}\Lambda}-\mathbf{p}_{\mathrm{K}}/2)^2}\\
&\times[\bar{u}(\mathbf{q}_{\mathrm{p}\Lambda}-\mathbf{p}_{\mathrm{K}}/2,\alpha_\Lambda)i\gamma^5\\
&\times\frac{g_{p\Lambda K^+}}{M_{\mathrm{p}}-\gamma^0\left(\sqrt{M^2_\Lambda+(\mathbf{q}_{\mathrm{p}\Lambda}-\mathbf{p}_{\mathrm{K}}/2)^2}+\sqrt{M^2_{\mathrm{K}}+\mathbf{p}^2_{\mathrm{K}}}\right)+\gamma\cdot(\mathbf{q}_{\mathrm{p}\Lambda}+\mathbf{p}_{\mathrm{K}}/2)}\,i\gamma^5u(\mathbf{p},\alpha_1)],
\end{aligned}
\tag{2}
$$

The wave functions of the particles in the transition $\mathrm{p+p\to p+\Lambda+K^+}$ are plane waves. In order to describe a physical reaction $\mathrm{p+p\to p+\Lambda+K^+}$, we suggest to take into account interactions between particles both in the final and in the initial state.

3 cross-sections for near threshold reactions $\mathrm{p+p\to p+\Lambda(\Sigma^0)+K^+}$

The contribution of the interaction in the pΛ-system can be obtained by summing up an infinite series of one-baryon loop diagrams with a point-like $(\bar{p}\bar{\Lambda})(p\Lambda)$ coupling describing a low-energy transition $\mathrm{p+\Lambda\to p+\Lambda}$ [7]. After the evaluation of momentum integrals and the renormalization of the wave functions of the proton and the Λ-hyperon [7] we can represent the contribution of this series in the phenomenological form in terms of the S-wave scattering length $a_{\mathrm{p}\Lambda}$ and the effective range $r_{\mathrm{p}\Lambda}$:

$$
f^{\mathrm{p}\Lambda\to\mathrm{p}\Lambda}(q_{\mathrm{p}\Lambda})=\frac{1}{1-\frac{1}{2}\,a_{\mathrm{p}\Lambda}r_{\mathrm{p}\Lambda}q^2_{\mathrm{p}\Lambda}+i\,a_{\mathrm{p}\Lambda}q_{\mathrm{p}\Lambda}}. \tag{8}
$$

This is the well-known Watson form for the final-state interaction [8] that has been used by Balewski *et al.* [9] for the description of the final pΛ interaction in the reaction $\mathrm{p+p\to p+\Lambda+K^+}$. Below we use the numerical values of the S-wave scattering length and the effective range, $a_{\mathrm{p}\Lambda}=-2.0\,\mathrm{fm}$ and $r_{\mathrm{p}\Lambda}=1.0\,\mathrm{fm}$, recommended by Balewski *et al.* [9]. We would like to emphasize that since finally the contribution of the final-state interaction is expressed in terms of phenomenological parameters, the S-wave scattering length and the effective range taken from experimental data, a knowledge of an explicit value of a coupling constant of a local $(\bar{p}\bar{\Lambda})(p\Lambda)$ interaction describing a low-energy transition $\mathrm{p+\Lambda\to p+\Lambda}$ and defining vertices in the one-baryon loop diagrams is not important [7]. For the analysis of elastic low-energy pΣ^0 scattering we assume that $a_{\mathrm{p}\Sigma^0}=a_{\mathrm{p}\Lambda}=-2.0\,\mathrm{fm}$ and $r_{\mathrm{p}\Sigma^0}=r_{\mathrm{p}\Lambda}=1.0\,\mathrm{fm}$. Below we show that this assumption does not contradict the experimental data [4].

Unlike the pΛ interaction in the final state, in order to describe the interaction in the initial pp state, we have to specify the coupling constant of the transition $\mathrm{p+p\to p+p}$. Since the relative momentum of the pp state is comeasurable with the proton mass, a Coulomb repulsion between protons can be neglected. We suggest to describe the pp interaction in the one-pion exchange approximation. As experimentally the relative momenta of the pp state differ slightly from the threshold momentum, we can represent the pp interaction describing the transition $\mathrm{p+p\to p+p}$ in the following local form:

$$
\mathcal{L}^{\mathrm{pp}\to\mathrm{pp}}(x)=\frac{1}{8}\,C_{\mathrm{pp}}\,[\bar{p}(x)p^c(x)][\bar{p^c}(x)p(x)]. \tag{9}
$$

This effective Lagrangian describes the pp system coupled in the spin-triplet state. In the factor 1/8 the multiplier 1/4 is caused by a Fierz transformation of the one-pion exchange interaction [7]. The phenomenological coupling constant C_{pp} is then defined by

$$
C_{\mathrm{pp}}=\frac{g^2_{\pi\mathrm{NN}}}{4\mathbf{p}^{\,2}}\ln\left(1+\frac{4\mathbf{p}^{\,2}}{M^2_\pi}\right). \tag{10}
$$

It is obtained from the one-pion exchange diagram of the transition $\mathrm{p+p\to p+p}$ by averaging over all possible directions of the relative momentum of the final pp state. We would like to accentuate that since relative momenta of the incident protons are very close to threshold, the quantity C_{pp} is practically constant determined by $\mathbf{p}^{\,2}\simeq p^2_0$ at $p_0=861.4\,\mathrm{MeV}$ or $p_0=917.5\,\mathrm{MeV}$ for the reactions $\mathrm{p+p\to p\,\Lambda+K^+}$ and $\mathrm{p+p\to p\,\Sigma^0+K^+}$, respectively.

By summing up an infinite series of one-proton loop diagrams the vertices of which are defined by the effective interaction eq. (9), we arrive at the expression [7]

$$[\bar{u^c}(-\mathbf{p},\alpha_2)u(\mathbf{p},\alpha_1)] \to \frac{[\bar{u^c}(-\mathbf{p},\alpha_2)u(\mathbf{p},\alpha_1)]}{1+\dfrac{C_{\rm pp}}{64\pi^2}\displaystyle\int\frac{d^4k}{\pi^2 i}\,{\rm tr}\Big\{\frac{1}{M_{\rm p}-\hat{k}}\frac{1}{M_{\rm p}-\hat{k}-\hat{P}}\Big\}}, \quad (11)$$

where $P = (2\sqrt{\mathbf{p}^2+M_{\rm p}^2},\mathbf{0})$.

After the subtraction of trivial **p**-independent divergent contributions and the renormalization of the wave functions of the protons we obtain the contribution of the interaction of the protons in the initial state

see equation (12) *on the next page*

where $C_{\rm pp}(\mathbf{p}^2,\Lambda)$ is given by

$$C_{\rm pp}(\mathbf{p}^2,\Lambda) = \frac{C_{\rm pp}}{1+\dfrac{C_{\rm pp}\mathbf{p}^2}{4\pi^2}\left[\ln\left(\dfrac{\Lambda}{M_{\rm N}}+\sqrt{1+\dfrac{\Lambda^2}{M_{\rm N}^2}}\right)-\dfrac{\Lambda}{\sqrt{M_{\rm N}^2+\Lambda^2}}\right]}. \quad (13)$$

The appearance of the cut-off Λ is caused by non-trivial **p**-dependent logarithmically divergent contributions. The cut-off Λ restricts from above 3-momenta of virtual proton fluctuations. Since our approach is an effective one, so the dependence of the amplitude on the cut-off seems to be usual [10,11]. The only thing one needs is to choose the value of the cut-off in an appropriate physical way. In our case it is reasonable to have Λ to be of order of the mass of the resonance nearest to the nucleon, that is the $\Delta(1232)$-resonance [6]. Therefore, in our calculations we would set $\Lambda = 1200\,{\rm MeV}$.

Thus, the amplitude of the reaction $\rm p + p \to p + \Lambda + K^+$ near threshold of the final $\rm p\Lambda K^+$ state is defined by

see equation (14) *on the next page*

where the last factor depending on $\alpha = 1/137$, the fine structure constant, and $q_{\rm pK^+}$, the relative momentum of the $\rm pK^+$ system, takes into account the Coulomb repulsion between the daughter proton and the $\rm K^+$-meson at low relative energies [9] (see also [7]), $M_{\rm pK^+} = M_{\rm p}M_{\rm K^+}/(M_{\rm p}+M_{\rm K^+})$ is the reduced mass of the $\rm pK^+$ system.

Then, relative momenta of the pp system are very close to threshold, $|\mathbf{p}| \simeq p_0 = 861.4\,{\rm MeV}$. Hence, we can calculate the contribution of the interactions in the pp state numerically. This gives

$$\frac{1}{1+\dfrac{C_{\rm pp}(p_0^2,\Lambda)}{8\pi^2}\dfrac{p_0^3}{\sqrt{p_0^2+M_{\rm p}^2}}\left[\ln\left(\dfrac{\sqrt{p_0^2+M_{\rm p}^2}+p_0}{\sqrt{p_0^2+M_{\rm p}^2}-p_0}\right)+\pi\, i\right]} = 0.308\,e^{-i\,46.6^0}. \quad (15)$$

For the reaction $\rm p + p \to p + \Sigma^0 + K^+$ we get $0.294\,e^{-i\,46.3^0}$.

By calculating numerically a part of the coupling constant $C_{\rm p\Lambda K^+}$ given by eq. (4), we reduce the amplitude of the reaction $\rm p + p \to p + \Lambda + K^+$ to the following form:

$$\begin{aligned}\mathcal{M}({\rm pp\to p\Lambda K^+}) &= 1.676\times10^{-8}\,e^{-i\,46.6^0}\\ &\times\frac{g_{\rm p\Lambda K^+}}{1-\frac{1}{2}\,a_{\rm p\Lambda}r_{\rm p\Lambda}q^2_{\rm p\Lambda}+i\,a_{\rm p\Lambda}q_{\rm p\Lambda}}\\ &\times[\bar{u}(-\mathbf{q}_{\rm p\Lambda}-\mathbf{p}_{\rm K}/2,\alpha_{\rm p})i\gamma^5u^c(\mathbf{q}_{\rm p\Lambda}-\mathbf{p}_{\rm K}/2,\alpha_\Lambda)]\\ &\times[\bar{u^c}(-\mathbf{p},\alpha_2)u(\mathbf{p},\alpha_1)]\\ &\times\sqrt{\frac{M_{\rm pK^+}}{q_{\rm pK^+}}\frac{2\pi\alpha}{e^{2\pi\alpha M_{\rm pK^+}/q_{\rm pK^+}}-1}}. \end{aligned} \quad (16)$$

For the reaction $\rm p + p \to p + \Sigma^0 + K^+$ the numerical factor is equal to $1.433\times10^{-8}\,e^{-i\,46.3^0}$.

The amplitude squared, averaged over polarizations of the initial protons and summed over polarizations of the final baryons amounts to

$$\begin{aligned}\overline{|\mathcal{M}({\rm pp\to p\Lambda K^+})|^2} &= 4.494\times10^{-15}\,p_0^2M_{\rm p}M_\Lambda\\ &\times\frac{g^2_{\rm p\Lambda K^+}}{\left(1-\frac{1}{2}\,a_{\rm p\Lambda}r_{\rm p\Lambda}q^2_{\rm p\Lambda}\right)^2+a^2_{\rm p\Lambda}q^2_{\rm p\Lambda}}\\ &\times\frac{M_{\rm pK^+}}{q_{\rm pK^+}}\frac{2\pi\alpha}{e^{2\pi\alpha M_{\rm pK^+}/q_{\rm pK^+}}-1}. \end{aligned} \quad (17)$$

For the reaction $\rm p + p \to p + \Sigma^0 + K^+$ the numerical factor acquires the value 3.285×10^{-15}.

The cross-section for the reaction $\rm p + p \to p + \Lambda + K^+$ reads

$$\sigma_{\rm p\Lambda K^+}(\varepsilon) = 0.043\,g^2_{\rm p\Lambda K^+}\,\varepsilon^2\,\Omega_{\rm p\Lambda K^+}(\varepsilon), \quad (18)$$

where the cross-section and the excess of energy $\varepsilon = 2\sqrt{\mathbf{p}^2+M_{\rm p}^2}-M_{\rm p}-M_\Lambda-M_{\rm K^+}$ are measured in (nb) and (MeV), respectively. The function $\Omega_{\rm p\Lambda K^+}(\varepsilon)$ related to the phase volume of the reaction is defined by

$$\begin{aligned}\Omega_{\rm p\Lambda K^+}(\varepsilon) &= \frac{1}{4\pi^3\varepsilon^2}\left(\frac{M_{\rm p}+M_\Lambda+M_{\rm K^+}}{M_{\rm p}M_\Lambda M_{\rm K^+}}\right)^{3/2}\\ &\times\int\frac{\delta^{(3)}(\mathbf{p}_{\rm p}+\mathbf{p}_\Lambda+\mathbf{p}_{\rm K})}{\left(1-\frac{1}{2}\,a_{\rm p\Lambda}r_{\rm p\Lambda}q^2_{\rm p\Lambda}\right)^2+a^2_{\rm p\Lambda}q^2_{\rm p\Lambda}}\\ &\times\frac{M_{\rm pK^+}}{q_{\rm pK^+}}\frac{2\pi\alpha}{e^{2\pi\alpha M_{\rm pK^+}/q_{\rm pK^+}}-1}\\ &\times\delta\left(\varepsilon-\frac{\mathbf{p}_{\rm p}^2}{2M_{\rm p}}-\frac{\mathbf{p}_\Lambda^2}{2M_\Lambda}-\frac{\mathbf{p}_{\rm K}^2}{2M_{\rm K^+}}\right)d^3p_{\rm K}d^3p_{\rm p}d^3p_\Lambda \end{aligned} \quad (19)$$

and normalized to unity at $\alpha \to 0$ and $a_{\rm p\Lambda} \to 0$.

The cross-section for the reaction $\rm p + p \to p + \Sigma^0 + K^+$ can be evaluated in analogy with the reaction $\rm p + p \to p + \Lambda + K^+$ and reads

$$\sigma_{\rm p\Sigma^0K^+}(\varepsilon) = 0.035\,g^2_{\rm p\Sigma^0K^+}\,\varepsilon^2\,\Omega_{\rm p\Sigma^0K^+}(\varepsilon). \quad (20)$$

$$[\bar{u^c}(-\mathbf{p},\alpha_2)u(\mathbf{p},\alpha_1)] \to \frac{[\bar{u^c}(-\mathbf{p},\alpha_2)u(\mathbf{p},\alpha_1)]}{1+\frac{C_{\mathrm{pp}}}{64\pi^2}\int\frac{\mathrm{d}^4k}{\pi^2 i}\,\mathrm{tr}\left\{\frac{1}{M_{\mathrm{p}}-\hat{k}}\frac{1}{M_{\mathrm{p}}-\hat{k}-\hat{P}}\right\}}$$
$$\to \frac{[\bar{u^c}(-\mathbf{p},\alpha_2)u(\mathbf{p},\alpha_1)]}{1+\frac{C_{\mathrm{pp}}(\mathbf{p}^2,\Lambda)}{8\pi^2}\frac{|\mathbf{p}|^3}{\sqrt{\mathbf{p}^2+M_{\mathrm{p}}^2}}\left[\ln\left(\frac{\sqrt{\mathbf{p}^2+M_{\mathrm{p}}^2}+|\mathbf{p}|}{\sqrt{\mathbf{p}^2+M_{\mathrm{p}}^2}-|\mathbf{p}|}\right)+\pi\, i\right]}, \tag{12}$$

$$\mathcal{M}(\mathrm{pp}\to \mathrm{p}\Lambda\mathrm{K}^+) = \frac{1}{2}\,C_{\mathrm{p}\Lambda\mathrm{K}^+}\,\frac{[\bar{u}(-\mathbf{q}_{\mathrm{p}\Lambda}-\mathbf{p}_{\mathrm{K}}/2,\alpha_{\mathrm{p}})i\gamma^5u^c(\mathbf{q}_{\mathrm{p}\Lambda}-\mathbf{p}_{\mathrm{K}}/2,\alpha_\Lambda)]}{1-\frac{1}{2}\,a_{\mathrm{p}\Lambda}r_{\mathrm{p}\Lambda}q_{\mathrm{p}\Lambda}^2+i\,a_{\mathrm{p}\Lambda}q_{\mathrm{p}\Lambda}}$$
$$\times\frac{[\bar{u^c}(-\mathbf{p},\alpha_2)u(\mathbf{p},\alpha_1)]}{1+\frac{C_{\mathrm{pp}}(\mathbf{p}^2,\Lambda)}{8\pi^2}\frac{|\mathbf{p}|^3}{\sqrt{\mathbf{p}^2+M_{\mathrm{p}}^2}}\left[\ln\left(\frac{\sqrt{\mathbf{p}^2+M_{\mathrm{p}}^2}+|\mathbf{p}|}{\sqrt{\mathbf{p}^2+M_{\mathrm{p}}^2}-|\mathbf{p}|}\right)+\pi\, i\right]}\sqrt{\frac{M_{\mathrm{pK}^+}}{q_{\mathrm{pK}^+}}\frac{2\pi\alpha}{e^{2\pi\alpha M_{\mathrm{pK}^+}/q_{\mathrm{pK}^+}}-1}}, \tag{14}$$

The function $\Omega_{\mathrm{p}\Sigma^0\mathrm{K}^+}(\varepsilon)$ results from eq. (19) via a replacement $\Lambda\to\Sigma^0$.

In terms of the axial-vector coupling constants D and F and $g_{\pi\mathrm{NN}}$ the coupling constants $g_{\mathrm{p}\Lambda\mathrm{K}^+}$ and $g_{\mathrm{p}\Sigma^0\mathrm{K}^+}$ are defined by [12]

$$g_{\mathrm{p}\Lambda\mathrm{K}^+} = -\frac{1}{\sqrt{3}}\left(\frac{D+3F}{D+F}\right)g_{\pi\mathrm{NN}},$$
$$g_{\mathrm{p}\Sigma^0\mathrm{K}^+} = -\left(\frac{D-F}{D+F}\right)g_{\pi\mathrm{NN}}. \tag{21}$$

The cross-sections eqs. (18) and (20) then read

$$\sigma^{\mathrm{pp}\to\mathrm{p}\Lambda\mathrm{K}^+}(\varepsilon) = 2.576\left(\frac{D+3F}{D+F}\right)^2\varepsilon^2\,\Omega_{\mathrm{p}\Lambda\mathrm{K}^+}(\varepsilon),$$
$$\sigma^{\mathrm{pp}\to\mathrm{p}\Sigma^0\mathrm{K}^+}(\varepsilon) = 6.208\left(\frac{D-F}{D+F}\right)^2\varepsilon^2\,\Omega_{\mathrm{p}\Sigma^0\mathrm{K}^+}(\varepsilon). \tag{22}$$

For the numerical analysis the functions $\Omega_{\mathrm{p}\Lambda\mathrm{K}^+}(\varepsilon)$ and $\Omega_{\mathrm{p}\Sigma^0\mathrm{K}^+}(\varepsilon)$ can be given in the more convenient form

$$\Omega_{\mathrm{p}\Lambda\mathrm{K}^+}(\varepsilon) = \frac{2}{\pi\varepsilon^2}\frac{M_{\mathrm{p}}+M_\Lambda}{M_\Lambda}\sqrt{\frac{M_{\mathrm{p}}M_\Lambda M_{\mathrm{K}^+}}{M_{\mathrm{p}}+M_\Lambda+M_{\mathrm{K}^+}}}$$
$$\times\int_0^\varepsilon\frac{1}{(1-a_{\mathrm{p}\Lambda}r_{\mathrm{p}\Lambda}M_{\mathrm{p}\Lambda}T_{\mathrm{p}\Lambda})^2+2a_{\mathrm{p}\Lambda}^2M_{\mathrm{p}\Lambda}T_{\mathrm{p}\Lambda}}$$
$$\times\int_{v_{\mathrm{pK}^+}^-}^{v_{\mathrm{pK}^+}^+}\frac{2\pi\alpha}{e^{2\pi\alpha/v_{\mathrm{pK}^+}}-1}\mathrm{d}v_{\mathrm{pK}^+}\mathrm{d}T_{\mathrm{p}\Lambda}, \tag{23}$$

where we have denoted

$$v_{\mathrm{pK}^+}^+ = \sqrt{\frac{2(M_{\mathrm{p}}+M_\Lambda+M_{\mathrm{K}^+})}{M_{\mathrm{K}^+}(M_{\mathrm{p}}+M_\Lambda)}\,(\varepsilon-T_{\mathrm{p}\Lambda})}+\sqrt{\frac{2M_\Lambda}{M_{\mathrm{p}}}\frac{T_{\mathrm{p}\Lambda}}{M_{\mathrm{p}}+M_\Lambda}},$$
$$v_{\mathrm{pK}^+}^- = \left|\sqrt{\frac{2(M_{\mathrm{p}}+M_\Lambda+M_{\mathrm{K}^+})}{M_{\mathrm{K}^+}(M_{\mathrm{p}}+M_\Lambda)}\,(\varepsilon-T_{\mathrm{p}\Lambda})}-\sqrt{\frac{2M_\Lambda}{M_{\mathrm{p}}}\frac{T_{\mathrm{p}\Lambda}}{M_{\mathrm{p}}+M_\Lambda}}\right|. \tag{24}$$

The function $\Omega_{\mathrm{p}\Sigma^0\mathrm{K}^+}(\varepsilon)$ should be obtained from the function $\Omega_{\mathrm{p}\Lambda\mathrm{K}^+}(\varepsilon)$ given by eq. (23) via a simple replacement $M_\Lambda\to M_{\Sigma^0}$. The numerical values of these functions are tabulated in tables 1 and 2, respectively.

For numerical calculations of the cross-sections we use $F=0.459\pm0.008$ and $D=0.798\pm0.008$ [13]. The numerical values of the cross-sections for the excess of energy ε ranging over the region $0.68\,\mathrm{MeV}\le\varepsilon\le 6.68\,\mathrm{MeV}$ [1,2,4] are adduced in tables 1 and 2.

4 Conclusion

We have developed a phenomenological approach to the description of the reactions $\mathrm{p+p\to p+\Lambda+K^+}$ and $\mathrm{p+p\to p+\Sigma^0+K^+}$ near thresholds of the final states $\mathrm{p\Lambda K^+}$ and $\mathrm{p\Sigma^0K^+}$, respectively.

The theoretical cross-section for the reaction $\mathrm{p+p\to p+\Lambda+K^+}$ agrees reasonably well with the experimental data. The numerical values of the cross-sections are adduced in table 1. We show that for the excess of energy ε ranging over the region $0.68\,\mathrm{MeV}\le\varepsilon\le6.68\,\mathrm{MeV}$ the cross-section is proportional to ε^2, $\sigma^{\mathrm{pp}\to\mathrm{p}\Lambda\mathrm{K}^+}(\varepsilon) = (4.5\pm0.1)\,\varepsilon^2\,\mathrm{nb}$. This fits well the experimental value $\sigma^{\mathrm{pp}\to\mathrm{p}\Lambda\mathrm{K}^+}(\varepsilon) = (4.4\pm0.7)\,\varepsilon^2\,\mathrm{nb}$ [2].

Table 1. Cross-sections for the reaction $p + p \to p + \Lambda + K^+$ for the excess of energy ranging over the region $0.68\,\mathrm{MeV} \le \varepsilon \le 6.68\,\mathrm{MeV}$. The experimental data are taken from ref. [2].

ε (MeV)	$\Omega_{p\Lambda K^+}(\varepsilon)$	$\sigma_{p\Lambda K^+}(\varepsilon)$ (nb)	$\sigma_{p\Lambda K^+}(\varepsilon)/\varepsilon^2$ (nb/MeV^2)	$\sigma_{p\Lambda K^+}(\varepsilon)_{\mathrm{exp}}$ (nb)	$\sigma_{p\Lambda K^+}(\varepsilon)_{\mathrm{exp}}/\varepsilon^2$ (nb/MeV^2)
0.68	0.516	1.8 ± 0.1	4.0 ± 0.1	2.1 ± 0.2	4.54
1.68	0.605	13.2 ± 0.3	4.7 ± 0.1	13.4 ± 0.7	4.75
2.68	0.616	34.1 ± 0.7	4.8 ± 0.1	36.6 ± 2.6	5.10
3.68	0.609	63.6 ± 1.2	4.7 ± 0.1	63.0 ± 3.1	4.65
4.68	0.594	100.3 ± 1.9	4.6 ± 0.1	92.2 ± 6.5	4.21
5.68	0.577	143.5 ± 2.8	4.5 ± 0.1	135 ± 11	4.18
6.68	0.560	192.6 ± 3.7	4.3 ± 0.1	164 ± 10	3.68
			4.5 ± 0.1		4.4 ± 0.7

Table 2. Cross-section for the reaction $p + p \to p + \Sigma^0 + K^+$ for the excess of energy ranging over the region $0.68\,\mathrm{MeV} \le \varepsilon \le 6.68\,\mathrm{MeV}$. The experimental data are taken from ref. [4].

ε (MeV)	$\Omega_{p\Sigma^0K^+}(\varepsilon)$	$\sigma_{p\Sigma^0K^+}(\varepsilon)$ (nb)	$\sigma_{p\Sigma^0K^+}(\varepsilon)/\varepsilon^2$ (nb/MeV^2)	$\sigma_{p\Sigma^0K^+}(\varepsilon)_{\mathrm{exp}}$ (nb)	$\sigma_{p\Sigma^0K^+}(\varepsilon)_{\mathrm{exp}}/\varepsilon^2$ (nb/MeV^2)
0.68	0.515	0.11 ± 0.01	0.23 ± 0.03	0.14 ± 0.06	0.29 ± 0.14
1.68	0.603	0.80 ± 0.10	0.27 ± 0.03	0.73 ± 0.34	0.26 ± 0.12
2.68	0.613	2.00 ± 0.25	0.28 ± 0.03	1.67 ± 0.77	0.23 ± 0.11
3.68	0.605	3.71 ± 0.46	0.28 ± 0.03	2.87 ± 1.32	0.21 ± 0.10
4.68	0.590	5.85 ± 0.72	0.27 ± 0.03	4.26 ± 1.97	0.20 ± 0.09
5.68	0.572	8.36 ± 1.03	0.26 ± 0.03	5.83 ± 2.69	0.18 ± 0.08
6.68	0.555	11.29 ± 1.38	0.25 ± 0.03	7.53 ± 3.47	0.17 ± 0.08
			0.26 ± 0.03		0.22 ± 0.11

The cross-section for the reaction $p + p \to p + \Lambda + K^+$ has been also measured for the higher excess of energies [4]: $\sigma^{pp\to p\Lambda K^+}(\varepsilon = 8.6\,\mathrm{MeV}) = (264 \pm 20)\,\mathrm{nb}$, $\sigma^{pp\to p\Lambda K^+}(\varepsilon = 10.9\,\mathrm{MeV}) = (392 \pm 33)\,\mathrm{nb}$ and $\sigma^{pp\to p\Lambda K^+}(\varepsilon = 13.2\,\mathrm{MeV}) = (534 \pm 47)\,\mathrm{nb}$. Our theoretical predictions for these energies read: $\sigma^{pp\to p\Lambda K^+}(\varepsilon = 8.6\,\mathrm{MeV}) = (298 \pm 6)\,\mathrm{nb}$, $\sigma^{pp\to p\Lambda K^+}(\varepsilon = 10.9\,\mathrm{MeV}) = (444\pm9)\,\mathrm{nb}$ and $\sigma^{pp\to p\Lambda K^+}(\varepsilon = 13.2\,\mathrm{MeV}) = (604\pm12)\,\mathrm{nb}$. For the calculation of these cross-sections we have also taken into account the momentum dependence of the structure function $C_{pp}(\mathbf{p}^2, \Lambda)$.

We would like to emphasize that within our approach the theoretical cross-section for the reaction $p + p \to p + \Lambda + K^+$ fits well the experimental values just at $\varepsilon = 138\,\mathrm{MeV}$. Really, at $\varepsilon = 138\,\mathrm{MeV}$ the theoretical value of the cross-section $\sigma^{pp\to p\Lambda K^+}(\varepsilon = 138\mathrm{MeV}) = (13.2 \pm 0.3)\,\mu\mathrm{b}$ agrees well with the experimental one $\sigma^{pp\to p\Lambda K^+}(\varepsilon = 138\,\mathrm{MeV}) = (12.0 \pm 0.4)\,\mu\mathrm{b}$ [3]. In average the accuracy of the agreement between the theoretical cross-section for the reaction $p + p \to p + \Lambda + K^+$ and the experimental data [2–4] is about 11%.

However, we cannot pass by the fact that the experimental value of the cross-section measured at $\varepsilon = 55\,\mathrm{MeV}$ [3]: $\sigma^{pp\to p\Lambda K^+}(\varepsilon = 55\,\mathrm{MeV}) = (2.7 \pm 0.3)\,\mu\mathrm{b}$ is smaller by a factor 1.7 than the theoretical one: $\sigma^{pp\to p\Lambda K^+}(\varepsilon = 55\,\mathrm{MeV}) = (4.7 \pm 0.3)\,\mu\mathrm{b}$. Since the theoretical cross-section in the excess of energy region $0.68\,\mathrm{MeV} \le \varepsilon \le 138\,\mathrm{MeV}$ is a smooth function of ε proportional to ε^2 and fits reasonably well the experimental value of the cross-section at $\varepsilon = 138\,\mathrm{MeV}$, we argue that the result obtained at $\varepsilon = 55\,\mathrm{MeV}$ seems to be underestimated and demands to be remeasured.

The cross-section for the reaction $p + p \to p + \Sigma^0 + K^+$ is also described well in our approach. The theoretical values of the cross-section adduced in table 2 are in reasonable agreement with the experimental data for all excess of energies from the interval $0.68\,\mathrm{MeV} \le \varepsilon \le 6.68\,\mathrm{MeV}$. In this energy region the theoretical cross-section is proportional to ε^2. The average value $\sigma^{pp\to p\Sigma^0K^+}(\varepsilon) = (0.26 \pm 0.03)\,\varepsilon^2\,\mathrm{nb}$ fits well the experimental data $\sigma^{pp\to p\Sigma^0K^+}(\varepsilon) = (0.22 \pm 0.11)\,\varepsilon^2\,\mathrm{nb}$ [4]. At $\varepsilon = 138\,\mathrm{MeV}$ we predict $\sigma^{pp\to p\Sigma^0K^+}(\varepsilon = 138\,\mathrm{MeV}) = (0.72 \pm 0.08)\,\mu\mathrm{b}$ that agrees well with the experimental value $\sigma^{pp\to p\Sigma^0K^+}(\varepsilon = 138\,\mathrm{MeV}) = (1.0 \pm 0.5)\,\mu\mathrm{b}$ [3].

In our approach the enhancement of the cross-section for the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Lambda+\mathrm{K}^+$ with respect to the cross-section for the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Sigma^0+\mathrm{K}^+$ is completely a unitary symmetry effect. In fact, the coupling constant $g_{\mathrm{p}\Sigma^0\mathrm{K}^+}$ is smaller by a factor of 0.16 than the coupling constant $g_{\mathrm{p}\Lambda\mathrm{K}^+}$. This is in agreement with the conclusion given by Kaiser [11]. However, unlike Kaiser's approach to the description of the reactions $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Lambda+\mathrm{K}^+$ and $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Sigma^0+\mathrm{K}^+$, we point out the dominant role of the contribution of the one-pion exchange and the strong interaction of the protons in the initial state.

The key-point of our approach to the description of the protons coupled in the initial state is a reduction of pp interaction to a local form via a phenomenological interaction eq. (10) based on the one-pion exchange. By having fixed the phenomenological coupling of a four-proton interaction, we have then succeeded in deriving the contribution of this interaction to the amplitudes of the reactions $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Lambda+\mathrm{K}^+$ and $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Sigma^0+\mathrm{K}^+$ via the summation of an infinite series of one-proton loop diagrams. After the evaluation of these diagrams and the renormalization of the wave functions of the protons, we have arrived at the expression that has turned out to be dependent on a cut-off Λ restricting from above the 3-momenta of the virtual proton fluctuations. The appearance of a dependence on the cut-off is usual in a phenomenological approach to the description of the reactions under consideration [10,11]. The main point is to fix this parameter in an appropriate way. The neglect of the contribution of baryon resonances to the amplitude of the pp interaction makes a hint that the cut-off Λ should be of the order of the mass of the nearest resonance, that is the $\Delta(1232)$-resonance. That is why we have set $\Lambda=1200\,\mathrm{MeV}$. As has been discussed above, this has led to the description of the cross-sections for the reactions $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Lambda+\mathrm{K}^+$ and $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Sigma^0+\mathrm{K}^+$ with an accuracy of about 11%.

We would like to underscore that our approach to the description of the protons coupled in the initial state is ideologically and technically rather similar to that applied by Achasov *et al.* [14] to the analysis of the contribution of the scalar $a_0(980)$- and $f_0(980)$-mesons treated as four-quark states [15] to the amplitudes of $\pi\pi$ and $\pi\mathrm{K}$ elastic scattering in the energy region of the order of 1 GeV.

Unlike other available theoretical approaches to the mechanism of $\Lambda\mathrm{K}^+$ and $\Sigma^0\mathrm{K}^+$ production in pp collisions [10,11,16], our mechanism does not demand the inclusion of exchanges of all mesons heavier than the π^0-meson and baryon resonances N(1650), N(1710) and so on. One can show that the summary contribution of the one-meson exchanges of $\eta(550)$- $\rho(770)$- and $\omega(780)$-meson and the scalar isoscalar meson $\sigma(700)$ [17–21] is of the order of 10% relative to the one-pion exchange. In fact, the estimate of the summary contribution of the $\eta(550)$-, $\sigma(700)$-, $\rho(770)$- and $\omega(780)$-meson exchanges relative to the one-pion exchange reads

$$\begin{aligned}
&\frac{1}{3}\left(\frac{D-3F}{D+F}\right)^2\frac{M_\pi^2+2M_\mathrm{p}(\sqrt{M_\mathrm{p}^2+p_0^2}-M_\mathrm{p})}{M_\eta^2+2M_\mathrm{p}(\sqrt{M_\mathrm{p}^2+p_0^2}-M_\mathrm{p})}\\
&-\frac{1}{g_\mathrm{A}^2}\frac{M_\pi^2+2M_\mathrm{p}(\sqrt{M_\mathrm{p}^2+p_0^2}-M_\mathrm{p})}{M_\sigma^2+2M_\mathrm{p}(\sqrt{M_\mathrm{p}^2+p_0^2}-M_\mathrm{p})}\\
&+2\,\frac{g_\rho^2}{g_{\pi\mathrm{NN}}^2}\frac{M_\pi^2+2M_\mathrm{p}(\sqrt{M_\mathrm{p}^2+p_0^2}-M_\mathrm{p})}{M_\rho^2+2M_\mathrm{p}(\sqrt{M_\mathrm{p}^2+p_0^2}-M_\mathrm{p})}\\
&=0.05-0.36+0.22=-0.09(9\%),
\end{aligned}$$

where we have assumed that the $\eta(550)$ is the eighth component of the octet of pseudoscalar mesons and the coupling constant of the σNN interaction is equal to $g_{\sigma\mathrm{NN}}=g_{\pi\mathrm{NN}}/g_\mathrm{A}$ [17] with $g_\mathrm{A}=1.267$, the axial-vector coupling constant [6]. Then, we have set $g_\rho=6.047$ as the $\rho\pi\pi$ coupling constant [6]. The value 9% can be reduced by including the contribution of the pseudoscalar $\eta'(958)$-meson. This confirms that with a good accuracy the one-pion exchange dominates in pp reactions for $\Lambda\mathrm{K}^+$ and $\Sigma^0\mathrm{K}^+$ production at thresholds of the final $\mathrm{p}\Lambda\mathrm{K}^+$ and $\mathrm{p}\Sigma^0\mathrm{K}^+$ states.

We do not take into account the contributions of baryon resonances N(1650), N(1710) and so on [16]. Nevertheless, the obtained agreement with the experimental data allows us to think that effectively the contributions of baryon resonances can be partly reproduced by the amplitude of the pp interaction in the initial state.

In our approach the daughter proton and the Λ-hyperon as well as the daughter proton and the Σ^0-hyperon are in the spin-singlet state. This implies that the direction of the spin of the Λ- and Σ^0-hyperons is strictly opposite to the direction of the spin of the daughter proton. Thereby, according to our approach, by measuring a polarization of the daughter proton, one measures unambiguously a polarization of the Λ- and Σ^0-hyperons. Of course, this is true only for an excess of energies very close to the thresholds of the final states $\mathrm{p}\Lambda\mathrm{K}^+$ and $\mathrm{p}\Sigma^0\mathrm{K}^+$. For an excess of energies at which the contribution of the spin-triplet state of the $\mathrm{p}\Lambda$ and $\mathrm{p}\Sigma^0$ system becomes perceptible the polarizations of the Λ- and Σ^0-hyperons are not so strictly determined. We are planning to carry out the analysis of the polarization properties of the Λ- and Σ^0-hyperons by taking into account the contribution of the spin-triplet states of the $\mathrm{p}\Lambda$ and $\mathrm{p}\Sigma^0$ systems in our forthcoming publications [2].

Recent measurements of the polarization of the Λ-hyperon in the reaction $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Lambda+\mathrm{K}^+$ at an excess of energy $\varepsilon=431\,\mathrm{MeV}$ [23] evidence an advantage of the K^+-meson exchange mechanism [24] with respect

[2] The most complete phenomenological analysis of the reactions $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Lambda+\mathrm{K}^+$ and $\mathrm{p}+\mathrm{p}\to\mathrm{p}+\Sigma^0+\mathrm{K}^+$ with polarized colliding protons near thresholds of the final states $\mathrm{p}\Lambda\mathrm{K}^+$ and $\mathrm{p}\Sigma^0\mathrm{K}^+$, respectively, has been carried out by Rekalo *et al.* [22].

to the one-pion exchange one. In our approach being valid for an excess of energies much less than $\varepsilon = 431\,\mathrm{MeV}$, the contribution of the K^+-meson exchange makes up about 0.1% in comparison with the exchange by the π^0-meson.

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